18. State and prove the Monodromy theorem.

19. State and prove Mittag-Leffler theorem.

20. Let Ω be a simply connected domain and $\Omega \neq \mathbb{C}$. Then prove that there exists a univalent function $f \in H(\Omega)$ such that $f(\Omega) = \Delta$. ie every simply connected domain which is a proper subset of C is conformally equivalent to the unit disk.

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NOVEMBER/DECEMBER 2024

GMA41/DMA41 — COMPLEX ANALYSIS – II

Time : Three hours Maximum : 75 marks

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SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Find the pole for $fz = \dfrac{1}{(z^3 - 1)(z+1)^2}$.

2. State Residue formula.

3. Write the formulas for $\cos n\theta$ and $\sin n\theta$ which occurs in the integrand.

4. For $\theta \in \left[0, \pi/2\right]$, prove that $\sin\theta \geq \left(2/\pi\right)\theta$.

5. What is analytic Continuation?

6. Define Singular point.

7. Prove that every mesomorphic function in C can be represented as a quotient of two entire functions.

8. State Zeta function.

9. Define equi continuous on Ω.

10. State Bloch's theorem.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL the questions.

11. (a) Find the residue at each of the isolated singularities of the function $\frac{z}{z^2 + 3z + 3}$.

Or

(b) State and prove Argument principle.

12. (a) Show that $I = \int_0^{2\pi} \cos^{2n}\theta \, d\theta = \frac{2\pi(2n)!}{2^{2n}(n!)^2}$.

Or

(b) Let $\gamma_\epsilon = \{Z : |Z - Z_o| = \epsilon$ and $\theta_1 k < \arg(Z - Z_o) \le \theta_2\}$. If F is continuous on $0 < |Z - Z_o| \le \epsilon$ and if $\lim_{Z \to Z_o}(Z - Z_o) f(z) = l$, then prove that $\lim_{\epsilon \to 0} \int_{\gamma C} f(z)\,dz = il(\theta_2 - \theta_1)$ where γ_ϵ is positively oriented.

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13. (a) Derive Poisson Integral formula.

Or

(b) State and prove Harnack's principle theorem.

14. (a) Prove that the series $\sum_{k=1}^{\infty} a_k$ converges absolutely iff the series $\sum_{k=1}^{\infty} \log(1 + a_k)$ converges absolutely.

Or

(b) State and prove Weierstrass factorization theorem.

15. (a) If f is a non-constant analytic function on a domain D, then prove that f is an open mapping.

Or

(b) State and prove Montel's theorem.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove Cauchy's Residue theorem.

17. Prove that $\int_0^{\infty} \frac{x^{m-1}}{(1 + x^n)}\,dx = \frac{\pi}{n \sin(m\pi/n)}$, for $m, n \in N$, with $n > m > 0$.